MAE 250D Final Project: The Standard Problem

Numerical Solution of Euler Equations in a Supersonic Engine Inlet via the MacCormack Finite Volume Method

Maziar Hemati

School of Engineering and Applied Sciences
University of California–Los Angeles
Los Angeles, California 90025

Friday, March 20, 2009
Supersonic engine inlet configuration

\[ M_1 = 2.9 \]
\[ p_1 = 10^5 \text{ Pa} \]
\[ \rho_1 = 1 \text{ kg/m}^3 \]

The gas is assumed to consist solely of air. Thus, the ratio of specific heats is taken as \( \gamma = 1.4 \).
**Governing Equations**

The two-dimensional Euler equations can be written in conservative form as

\[
\frac{\partial U}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = 0
\]

where

\[
U = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix}, \quad E = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (e + p)u \end{bmatrix}, \quad F = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ (e + p)v \end{bmatrix}.
\]

The Euler equations are subject to the boundary conditions of no-flow through at the wall surfaces.
Numerical Approach

Grid Generation

Figure: Grid generated using $IL = 42$, $IS = 6$, $JL = 22$
Numerical Approach
MacCormack Method with Artificial Dissipation

Predictor:

\[
U_{ij}^{n+1} = U_{ij}^n - \frac{\Delta t}{V_{ij}} \left[ \frac{\hat{E}_{i+1j} | S'_{i+\frac{1}{2}} |}{2} + \frac{\hat{E}_{ij} | S'_{i-\frac{1}{2}} |}{2} + \frac{\hat{F}_{ij+1} | S'_{j+\frac{1}{2}} |}{2} + \frac{\hat{F}_{ij} | S'_{j-\frac{1}{2}} |}{2} \right]
\]

Corrector:

\[
U_{ij}^{n+1} = \frac{1}{2} \left\{ U_{ij} + U_{ij}^{n+1} - \frac{\Delta t}{V_{ij}} \left[ \frac{\hat{E}_{i+1j}^{n+1} | S'_{i+\frac{1}{2}} |}{2} + \frac{\hat{E}_{i-1j}^{n+1} | S'_{i-\frac{1}{2}} |}{2} \right. \right.
\]
\[+ \frac{\hat{F}_{ij+1}^{n+1} | S'_{j+\frac{1}{2}} |}{2} + \frac{\hat{F}_{ij-1}^{n+1} | S'_{j-\frac{1}{2}} |}{2} \left\} \right\}
\]
Numerical Approach
MacCormack Method with Artificial Dissipation

Sample Dissipation Term:

\[ \hat{E}^{n}_{ij} = E^{n}_{ij} - \epsilon \left( |u'| + c \right)^{n}_{ij} \frac{|p^{n}_{ij} - 2p^{n}_{i-1j} + p^{n}_{i-2j}|}{p^{n}_{ij} + 2p^{n}_{i-1j} + p^{n}_{i-2j}} (U^{n}_{ij} - U^{n}_{i-1j}) \]

Note:

\[ p^{n}_{0j} = 2p^{n}_{1j} - p^{n}_{2j} \]
\[ p^{n}_{i0} = 2p^{n}_{i1} - p^{n}_{i2} \]
Numerical Approach

Numerical Boundary Conditions

**Inlet:** Since the flow at the inlet is supersonic ($M_1 > 1$) in this configuration, $U = U_1$ must be specified.

**Exit:** The value of $U$ is determined by conditions downstream.

Linear extrapolation

$$U = U_{IL} = 2U_{IL-1} - U_{IL-2}.$$
**Walls: Zeroth Order Extrapolation**

\[
\rho_{\text{wall}} = \rho_{\text{wall}+1}
\]

\[
\rho_{\text{wall}} = \rho_{\text{wall}+1}
\]

\[
u_{\text{wall}} = u_{\text{wall}+1} \cos(2\alpha) + v_{\text{wall}+2} \sin(2\alpha)
\]

\[
v_{\text{wall}} = u_{\text{wall}+1} \sin(2\alpha) - v_{\text{wall}+2} \cos(2\alpha)
\]
IL = 42

IS = 6

JL = 22

$\epsilon = 0.1$

$|| \frac{\Delta p^n - p^{n-1}}{p^{n-1}} ||_\infty = 10^{-6}$
Computational Results

Pressure

![Pressure Graph](image-url)
Computational Results

Mach Number

![Mach Number Contour Plot]

- x-location (m)
- y-location (m)
- Mach Number

<table>
<thead>
<tr>
<th>x-location (m)</th>
<th>y-location (m)</th>
<th>Mach Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>0</td>
<td>3.0</td>
</tr>
<tr>
<td>0</td>
<td>0.2</td>
<td>2.5</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4</td>
<td>2.0</td>
</tr>
<tr>
<td>1.0</td>
<td>0.6</td>
<td>1.5</td>
</tr>
<tr>
<td>1.5</td>
<td>0.8</td>
<td>1.0</td>
</tr>
<tr>
<td>2.0</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>2.5</td>
<td>1.2</td>
<td>0.0</td>
</tr>
<tr>
<td>3.0</td>
<td>1.4</td>
<td>-0.5</td>
</tr>
<tr>
<td>3.5</td>
<td>1.6</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

Discussion of Results
Computational Results

Density

![Density Contour Plot]

The figure shows the density distribution for a given section of the problem, with color representing different density values. The x-location and y-location axes are marked, and the density is given in kg/m$^3$. The color bar indicates the range of density values from 1 to 4 kg/m$^3$. The grid lines help visualize the spatial variation of density across the x and y dimensions.
Discussion of Results

Pressure

This comparison is made along the vertically centered grid-line (i.e. \( j = 11 \)).
Discussion of Results

Mach Number

This comparison is made along the vertically centered grid-line (i.e. $j=11$).
Discussion of Results

Density

![Density Graph]

This comparison is made along the vertically centered grid-line (i.e. j=11).