Reduced Order Modeling of Turbulent Flows

MAE 252B: Final Presentation

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# Reduced Order Models
- What are they?
- Why do we care?

# Flow Control
- An overview
- An example of turbulence control

# Concluding Remarks
The Navier-Stokes equations can be expressed differently based on flow conditions.

Examples:

1. Potential Flow Analysis
2. Vortex Methods
3. Boundary Layer Equations
4. Euler Equations
5. RANS
6. LES
Mathematical ROM

Now instead of neglecting “higher-order” physical quantities, we neglect “higher-order” basis functions.

**Objective:**
Capture the essential physics at play, while reducing the expense of solving the governing equations.

ROMs require the solution of the flow-field *a priori* and do not capture *all* of the flow physics. As such, they are a poor choice for simulations.
On the Meaning of “Low-Dimensional”

We wish to represent an infinite-dimensional PDE as a low-dimensional system of ODEs.

**NOTE: Low-Dimensional**

“Low” in the context of fluid mechanics, *not* dynamical systems.

For example:

- Low-dimensional model $\sim (10 - 100)$
- Turbulent flow representation $\sim (\text{Re}^\frac{9}{4})$
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Applications
Understanding Fundamental Mechanisms of Turbulence

Motivation:
1. No closed form solution of Navier-Stokes in $\mathbb{R}^3$ exists.
2. Low-dimensional models can be analyzed using dynamical systems theory.

Useful in the study of:
1. Coherent structures
2. Bursting and sweeping processes
3. Simple flows
   - jets
   - wakes
4. shear layers
5. boundary layers
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Turbulent Flow Control

Motivation:
1. Control system design models need only an input-output description of the flow.
2. Low-dimensional models are computationally inexpensive.
3. Many aspects of the flow we wish to control can be analyzed by the methods discussed here.

Useful for:
1. Controller design
2. Estimator design and implementation
3. Flow-field reconstruction
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Definition: *Flow Control*

The notion of achieving a desired objective as a function of space and time by some means of flow manipulation.

Classifications:

1. Passive
2. Active
   - Open-Loop
   - Closed-Loop
Flow Control

Control for Drag Reduction (Prabhu et al. 2001)

Consider:

- Turbulent flow between parallel walls.
  - Fully developed
  - Incompressible
- Active control using wall-transpiration.
  - Opposition control
  - Optimal control
Some notation before we get started:

All quantities are nondimensionalized using:
- Reference length scale: channel half-width, $\delta^*$
- Reference velocity: friction velocity, $u_\tau^* = (\tau_w^* / \rho^*)^{1/2}$

**Note:** $\tau_w^*$ is the average wall shear-stress and an asterisk (*) denotes *dimensional* quantities.

Derived quantities:
- Convective time scale: $t = \delta^* / u_\tau^*$
- Reynolds number: $Re_\tau = u_\tau^* \delta^* / \nu^*$
- Wall units: $y^+ = y Re_\tau$ and $t^+ = t Re_\tau$
Flow Control
Control for Drag Reduction (Prabhu et al. 2001)

**Opposition Control**
Sense the presence of near-wall coherent structures and suppress them by opposing their motion through suction or blowing.

Wall sensing is performed at $y^+ = 16$.

**Optimal Control**
Minimize a cost-functional that targets the turbulent kinetic energy at the end of a time window $T$.

Requires complete knowledge of the flow-field. The time window used is $T^+ = 36$. 
Flow Control
Control for Drag Reduction (Prabhu et al. 2001)

Drag histories for turbulent channel flow
\((Re_T = 180)\)

- **Opposition control**: 25% drag reduction
- **Optimal control**: 40% drag reduction

Courtesy of Prabhu et al. (2001).
Concluding Remarks

Reduced order models do not provide a complete solution to the “problem of turbulence.”

But, as we have seen, they provide a useful approach for:

- Analyzing the behavior of turbulent flows with predominant coherent structures;
- Creating efficient and acceptable models for flow control and estimation.
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